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The work covered in this report involves a continuation of previous investigations of the Ferraro collisionless plasma model,[1] an effect due to radiation reaction in a collisionless plasma, and the description of a collisionless plasma model which utilizes the DeBye screening distance to construct constitutive relations between the electric and displacement fields and the magnetic and induction fields.

In the general three dimensional Ferraro model one has three Euler equations for the bulk flow velocity, three for ion-electron relative velocity, a particle pair number continuity equation and a charge continuity equation. The Maxwell equations for curl $\underline{E}$ and curl $\underline{H}$ involve twelve variables. Constitutive relations reduce these to six independent variables. The assumption of quasi-neutrality leads one to require that the solutions satisfy div $\underline{D}=0$, where $\underline{D}$ is the displacement field. In addition one also requires div $\underline{B}=0$, where $\underline{B}$ is the magnetic induction field. Previous work on the model[2] assumed the constitutive relations $\underline{B}=\underline{H}$, and $\underline{D}=\underline{E}$. However, the solutions obtained did not satisfy the condition div $\underline{E}=0$. Therefore, it would seem to be of interest to see if there exist solutions to the model which satisfy the quasi-neutrality condition as well as $\underline{B}=\underline{H}$. We have shown that such solutions exist by exhibiting three of them. The solutions are exact, two of them being both space and time dependant and the other being steady state. The solutions are three dimensional, but depend on only one of the space coordinates. These solutions describe waves propagating without dispersion at a single frequency and wavelength. The propagation velocity in each case is equal to the non-oscillating component of the flow velocity, which is constant. The oscillations are transverse to the latter component of the flow velocity. Assuming a proton-electron plasma, the propagation speed in the steady state solution is equal to the Alfvén speed determined by the constant, non-oscillating magnetic field component parallel to the propagation velocity and the sum of the two particle masses. For non-relativistic flow velocities, the propagation speed in one of the non-steady solutions is approximately half the latter Alfvén speed while in the other non-steady solution, its square is approximately half the square of the Alfvén speed. The lack of dispersion arises because of the insistence on satisfying the subsidiary conditions that div $\underline{E}$ and div $\underline{B}$ are zero and certain special conditions.

We have also developed exact two dimensional steady state solutions which satisfy div $\underline{E}=0$, but not $\underline{B}=\underline{H}$. The latter solutions are valid only over a finite region. Since it has never been demonstrated (for example, from a microscopic model) that the macroscopic constitutive relations, $\underline{B}=\underline{H}$, should obtain in a very rare (10-100 particles per cubic centimeter) collisionless plasma, such solutions may be significant.

In addition we have exact two dimensional solutions for which div $\underline{E}$ does not vanish, but $\underline{B}=\underline{H}$. However, for sufficiently small fields, the ratio of div $\underline{E}$ and the pair density will be small compared with unity.

The former group of two dimensional solutions involves open streamlines, while the latter group has closed streamlines. All three groups of the above mentioned solutions are oscillatory. For typical solar wind parameters, the characteristic oscillation length will be of the order of several kilometers. If the models considered are physically significant, it is suggested that perhaps the oscillatory behavior may be an inherent dynamical property of solar wind flow.

In view of the fact that we have raised the question of whether or not the macroscopic constitutive relations, $\underline{E}=\underline{D}$ and $\underline{B}=\underline{H}$ are valid in a rare collisionless plasma, we have studied a model which is based on the assumption that only those particles which are within a DeBye length of a particle effectively interact with it. The model then allows for polarization and magnetization. Our investigation of the model at present, however, indicates that the equations may be put in such a form as to yield essentially the same solutions as the Ferraro model.

An effect which may not be observable in plasma flow past an obstacle, but might still effect on the consideration of certain approximations is that of radiation reaction. Since the radiation reaction is different for protons and electrons it will introduce a small charge separation effect. This allows one an alternative justification for the neglect of the non-vanishing of div $\underline{E}$ in such calculations as those of Adlam and Allen and Davis et al.[2] The radiation reaction gives rise to a small current parallel to the flow velocity. The latter current may be neglected in the Euler equations and the violation of the assumption of quasi-neutrality will also be negligible.

The above outline of work performed represents a brief description of material to be presented in four papers.

REFERENCES

1. V. C. A. Ferraro, J. Geophys. Res. 57, 15(1952).

   For definition of the model appropriate to the present discussion see J. H. Adlam and J. E. Allen, Phil. Mag. 3, 448(1958).

2. See for example, J. H. Adlam and J. E. Allen, Phil. Mag. 3, 448(1958).

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